

# Top-Down SBP: Turning Graph Clustering Upside Down

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FRANK WANYE
VITALIY GLEYZER
EDWARD KAO
WU-CHUN FENG

JULY 22, 2025 34TH ACM INTERNATIONAL SYMPOSIUM ON HIGH-PERFORMANCE PARALLEL AND DISTRIBUTED COMPUTING (HPDC)



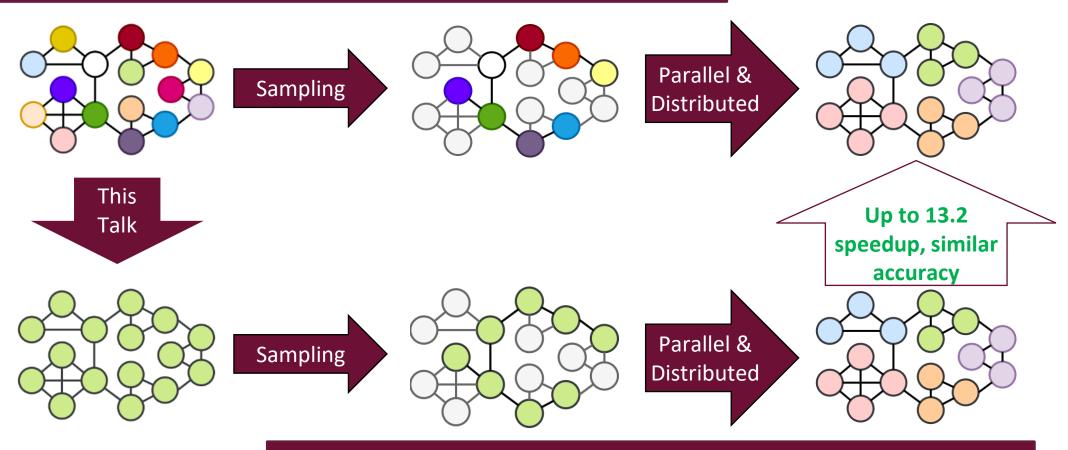




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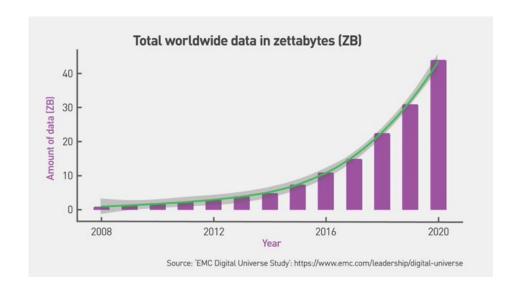
### Forecast

### Bottom-Up Stochastic Block Partitioning (SBP)

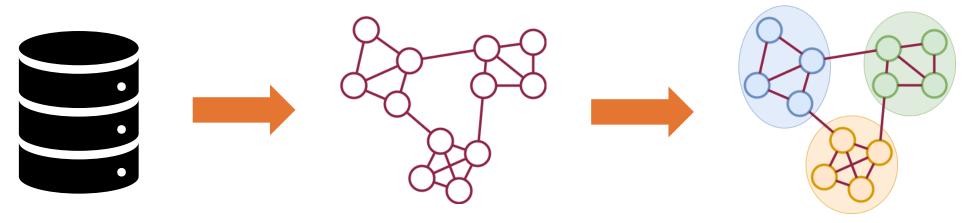


Top-Down Stochastic Block Partitioning (SBP)

### Introduction



- Worldwide data collected doubling every 2 years
- Much of this data is relational  $\rightarrow$  graph representation
- Groups of strongly connected vertices correlate to functional groups within data
- Graph clustering: process of finding such groups



### Motivation

### Applications across many domains









- Accurate graph clustering is difficult
- Difficulty highlighted by collaborative efforts, e.g., Graph Challenge [1], sponsored by IEEE/Amazon/MIT

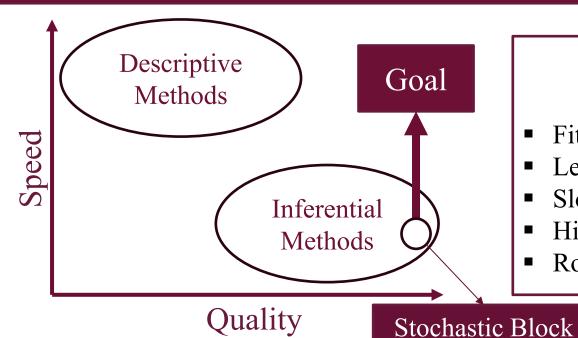
### Fast vs. Accurate Graph Clustering

- Optimal graph clustering is NP-hard  $\rightarrow$  solved via heuristics
- Two classes of heuristics: descriptive and inferential<sup>[1]</sup>

### Enable <u>accurate</u> graph clustering in large graphs by accelerating inferential graph clustering methods

#### Descriptive Methods

- **Encompass many** commonly used heuristics
- Fast
- Lower quality solutions
- Prone to overfitting



#### Inferential Methods

- Fit statistical models to data
- Less commonly used
- Slow
- Higher quality solutions
- Robust against overfitting

Partitioning (SBP)

### OVERVIEW

- 1. Introduction to Graph Clustering
- 2. Background & Contributions
- 3. Approach
  - a) Top-Down SBP
  - b) Accelerated Top-Down SBP
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### SBP Algorithm Overview

Statistical Model

Quality Function

Optimization Algorithm

### Stochastic Blockmodel (SBM)<sup>[1]</sup>

- Generative model
- Models the graph in relation to connectivity between clusters (blocks)

		Cluster			
		0	1	2	3
	0	12	2	3	1
Cluster	1	1	20	2	4
Clu	2	3	1	13	5
	3	2	4	1	17
			<b>A</b>		

Number of edges from cluster 3 to cluster 1

### **Description Length (H)**<sup>[2]</sup>

- Quality function for inference over **SBM**s
- Number of bits needed to encode SBM
- $H = f(graph \ size, blockmodel \ parameters)$
- Lower H =
  - Better compression
  - Lower entropy → more stability
  - Better quality of clusters

### SBP Algorithm Overview

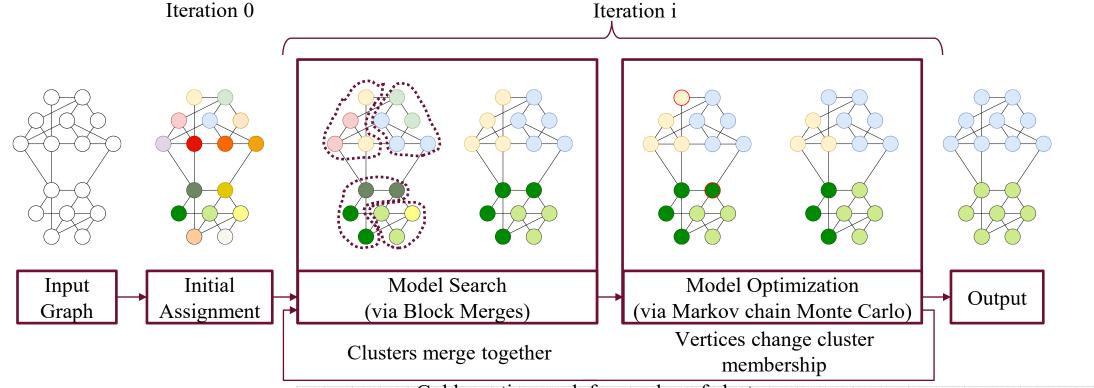
Statistical Model

Quality Function

Optimization Algorithm

Iterative, agglomerative, Markov chain Monte-Carlo (MCMC) based optimization of description length  $H^{[1,2]}$ 

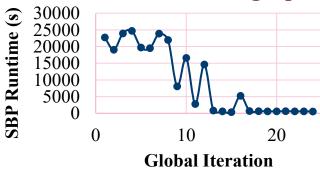
 $O(E \log^2 E)$ E: number of graph edges



### Contributions

### **Computational Profile**

### Runtime Breakdown by Iteration on 1M vertex graph

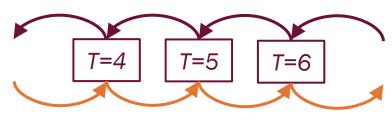


Challenges

- Random memory access patterns
- Row *and* column-wise indexing
- Front-heavy computation
- Top-Down computation approach
  - 7.7X speedup over Bottom-Up
  - 4X lower memory usage

### **MCMC Computation**

#### **Dependencies**



**Execution** 

- Inherently sequential optimization technique<sup>[1]</sup>
- State at time T depends on *all* previous timesteps
- Integration of Top-Down approach with prior SBP parallelization efforts
  - 13.2X speedup over accelerated Bottom-Up

#### **Contributions**

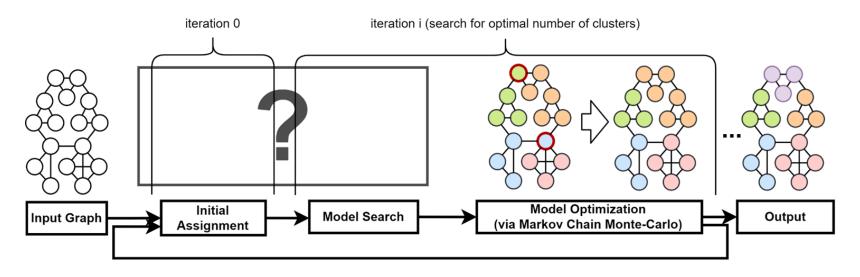
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### Top-Down SBP: Overview

### **Approach**

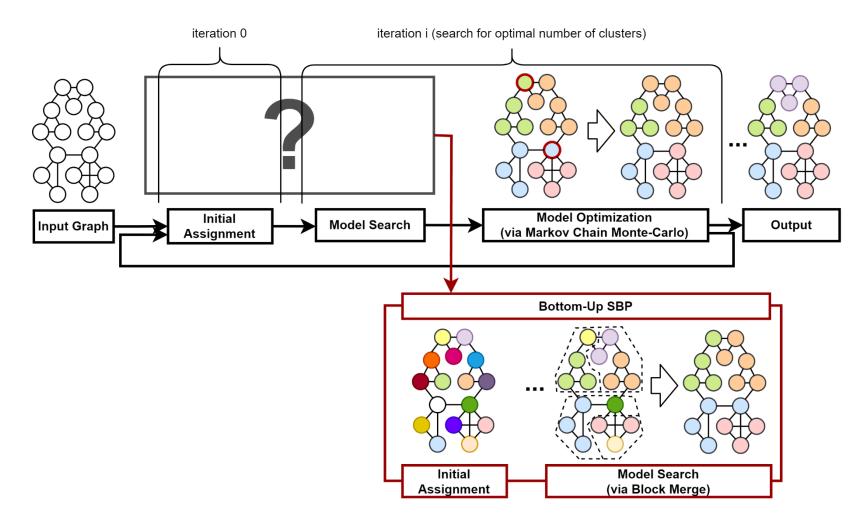
- Replicate overall algorithm structure of SBP
- Block merges replaced with block splits
- Splits accepted/rejected based on change in SBM description length
- Same algorithmic complexity: O(E log<sup>2</sup> E)



### Top-Down SBP: Overview

### **Approach**

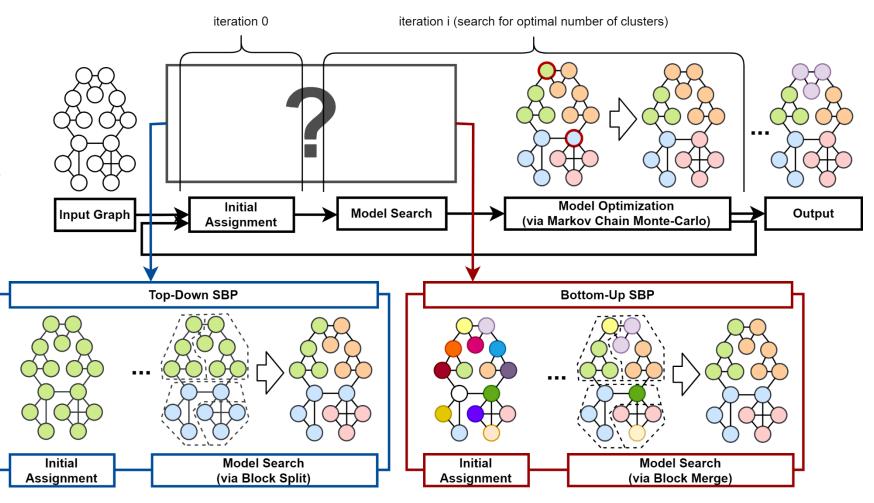
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### Top-Down SBP: Overview

### **Approach**

- Replicate overall algorithm structure of SBP
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### Block-Splitting Heuristic

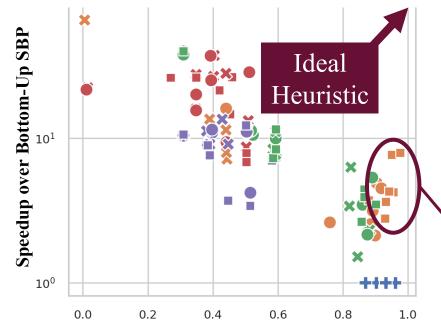
#### **Splitting Heuristics**

- Uniform random
- Two competing snowball samples
- One snowball sample
- Snowball sampling based on connectivity

#### **Split Initializations**

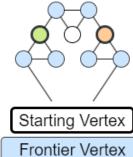
- Uniform random
- Degree-weighted random initialization
- Selecting the two highest-degree vertices

### **Exploring Splitting Heuristics**

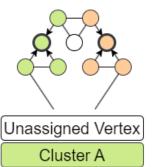


**Accuracy (Normalized Mutual Information)** 

Step 1: Select Two Starting Vertices For New Clusters



Step 2:
Assign Frontier Vertices
to Clusters Based on
Connectivity



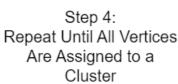
Cluster B

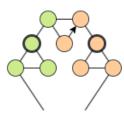


Step 3:

Identify New

Frontier Vertices





Cluster A
Cluster B

### **Best Heuristic: Connectivity snowball + random initialization**

- Idea: clusters should be split based on vertex locality
- Two vertices are randomly chosen to initialize the new clusters

### Single-Threaded Results

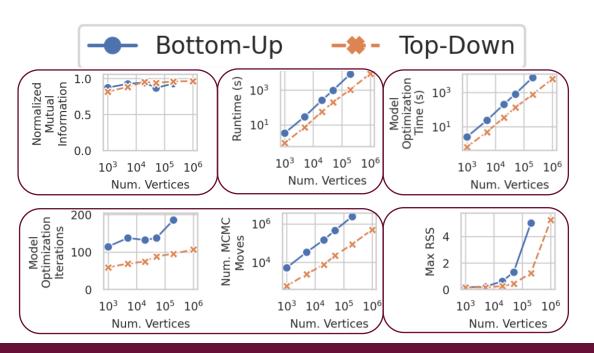
### **Graphs**

 Official IEEE/Amazon/MIT Graph Challenge synthetic datasets

Num. Vertices	Num. Edges	Num. Clusters
1,000	8,032	11
5,000	51,157	19
20,000	473,329	32
50,000	1,187,682	44
200,000	4,754,406	71
1,000,000	23,772,977	125

#### **Experiments**

- Running single-threaded Bottom-Up SBP and Top-Down SBP
- Ookami cluster
  - 32 GB memory, Fujitsu A64FX CPUs



Little-to-no difference in accuracy

Up to 7.7X speedup over Bottom-Up SBP

Fewer MCMC iterations and fewer MCMC vertex moves

results to the second of the second

Up to 4.1X lower memory usage

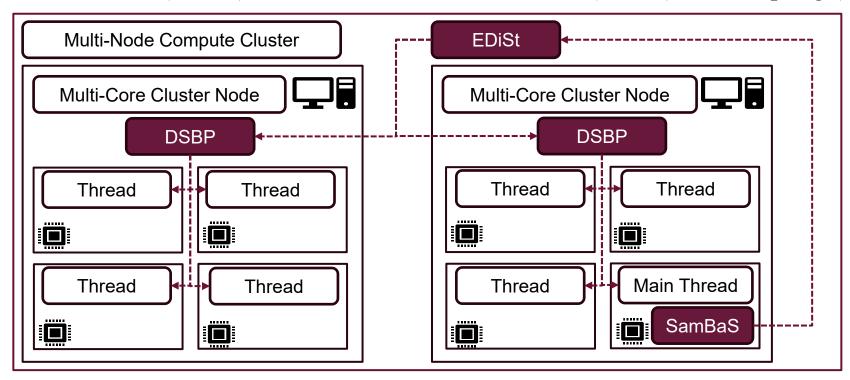
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### Accelerated Top-Down SBP: Overview

• Bottom-Up SBP has been successfully accelerated using a combination of the following<sup>[1][2]</sup>:

Shared-Memory Parallelization (**DSBP**)<sup>[2]</sup> + Multi-Node Parallelization (**EDiSt**)<sup>[3]</sup> + Sampling (**SamBaS**)<sup>[4]</sup>



• We adapt these approaches from Bottom-Up SBP to Top-Down SBP

<sup>[1]</sup> Frank Wanye, Vitaliy Gleyzer, Edward Kao, Wu-chun Feng. An Integrated Approach for Accelerating Stochastic Block Partitioning. In Proceedings of the 27th IEEE High Performance Extreme Computing Conference (HPEC), 2023.

<sup>[2]</sup> Ahsen Uppal, Thomas Rolinger, Howie Huang. Decontentioned Stochastic Block Partition. In Proceedings of the 27th IEEE High Performance Extreme Computing Conference (HPEC), 2023.

<sup>17 /</sup> APPROACH

<sup>[3]</sup> Frank Wanye, Vitaliy Gleyzer, Edward Kao, Wu-chun Feng. Exact Distributed Stochastic Block Partitioning. In Proceedings of the 25th IEEE International Conference on Cluster Computing (CLUSTER), 2023.

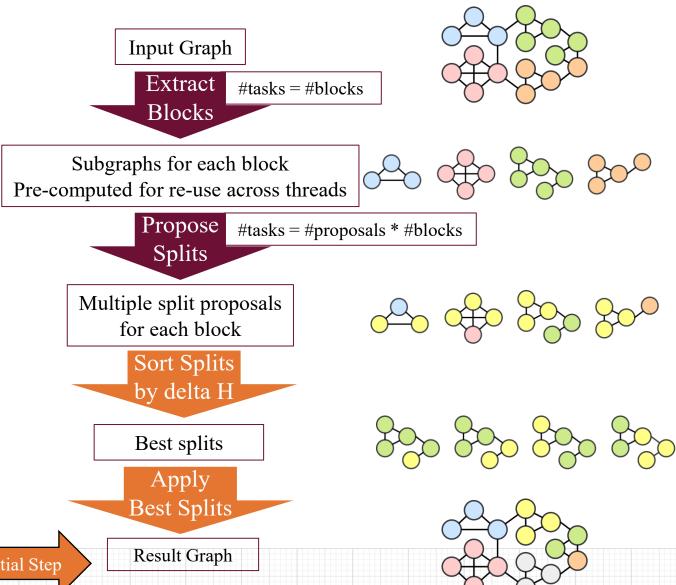
### Shared-Memory Parallelism

#### **Model Optimization Phase**

- Batched asynchronous Gibbs method
  - Embarrassingly parallel within each batch

#### **Model Search Phase**

Requires pre-computing subgraphs to reduce memory usage



### Multi-Node Parallelism

### **EDiSt:** Exact Distributed **Stochastic Block Partitioning**

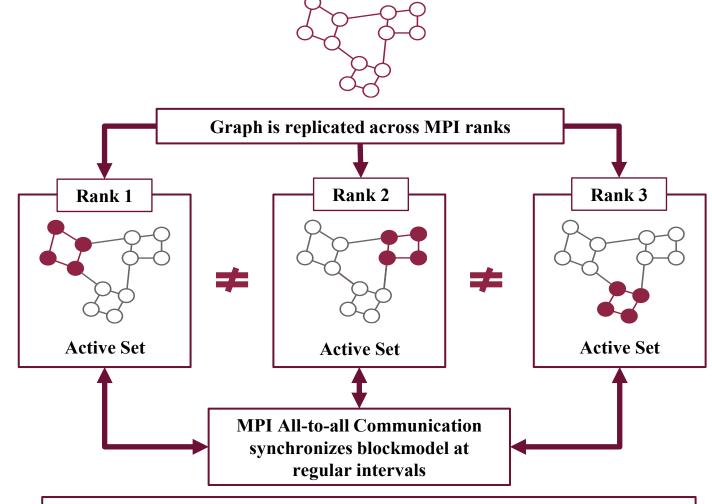
- Data replication → minimize broken dependencies → helps retain accuracy
- Difference between Bottom-Up and Top-Down SBP: amount of communication in model search phase

#### **Bottom-Up SBP**

• Block-level operation: O(#blocks) data transferred

#### **Top-Down SBP**

• Vertex-level operation: O(#vertices) data transferred

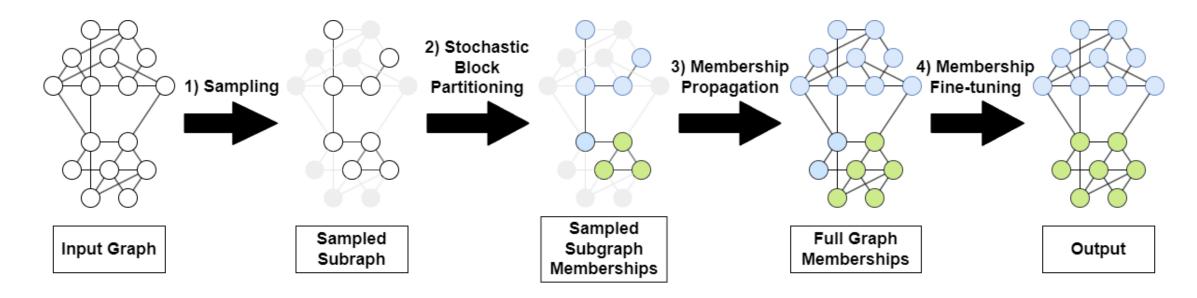


Active sets chosen so as to equalize a) number of vertices and b) total vertex degrees across MPI ranks for load balancing

### Sampling

**SamBaS:** 4 step sampling approach to accelerating SBP

Integration: Run Top-Down SBP in step 2



- ✓ Reduces memory and compute cost of initial iterations
- ✓ Flexible → SBP & fine-tuning can be replaced with accelerated variants

### Accelerated Top-Down SBP Results

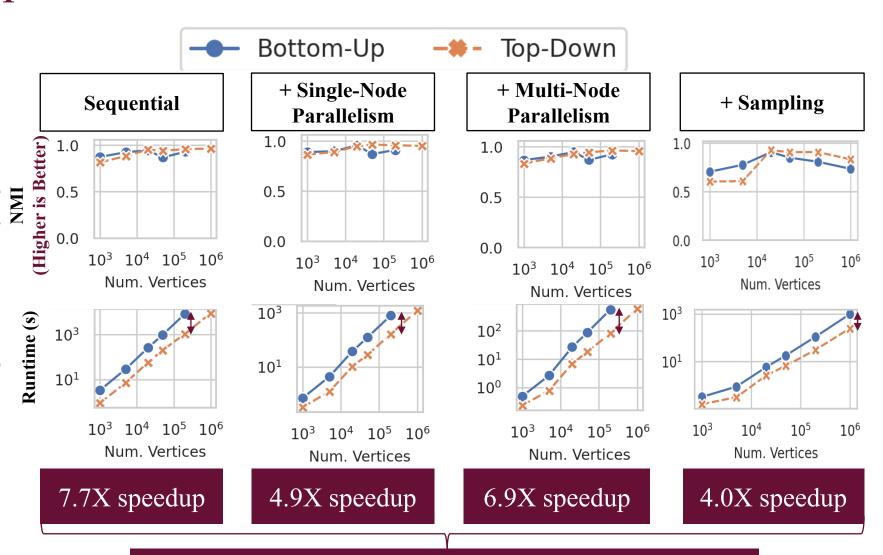
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#### Hardware

Ookami cluster: 1-4 nodes, 48 cores per node, 32 GB memory, Fujitsu A64FX CPUs



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### Top-Down SBP: Real-World Graph Results

Dataset Name	Num. Vertices	Num. Edges
cit-HepPh	34,546	421,534
soc-Slashdot0902	82,168	870,161
web-BerkStan	685,230	7,600,595
amazon0601	403,394	10,162,164
citPatents	3,774,768	16,518,947
cu-2005	862,664	18,733,713
wiki-topcats	1,791,489	28,508,141
wikipedia-20070206	3,515,067	45,013,315

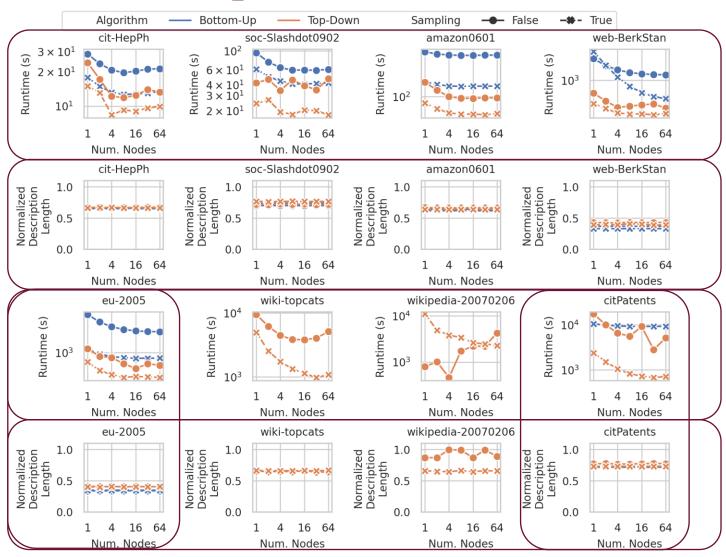
#### Hardware

 Ookami (1-64 nodes, 48 cores per node, 32 GB memory, Fujitsu A64FX CPUs)

#### Little-to-no difference in accuracy

Accelerated Top-Down SBP is up to 13.2X faster than equivalent Bottom-Up SBP, and up to 403X faster than sequential Bottom-Up SBP

Process graphs up to 4.4X larger on same hardware

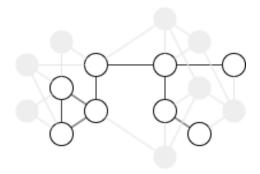


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### Future Work

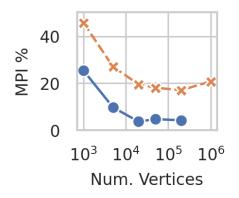
#### Sampling



Limitation: Large (30% - 50%) sample sizes needed to maintain accuracy

Potential Solution: Alternative data reduction methods like coresets could help reduce required sample size

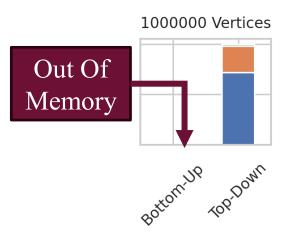
#### **Parallelism**



**Limitation:** Poor parallel efficiency

Potential Solution: MPI all-toall primitives → MPI singlesided primitives GPU acceleration

#### **Memory Usage**



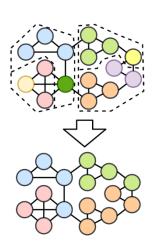
**Limitation:** High memory usage limits graph size

Potential Solution: Data distribution in multi-node implementation could alleviate memory bottlenecks

### Summary

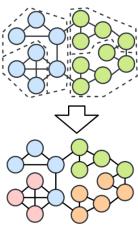
### **Bottom-Up Clustering**

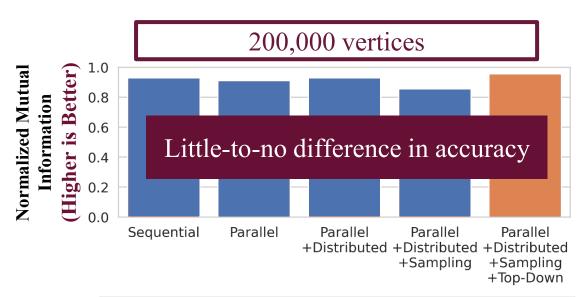
- Traditional approach to SBP
- Clusters merge over time
- **✗** High memory requirements
- **★** Many MCMC moves
- Slow compute

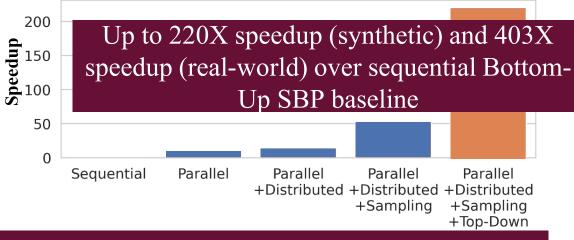


### **Top-Down Clustering**

- Novel approach
- Clusters split over time
- ✓ Lower memory requirements
- ✓ Fewer MCMC moves
- ✓ Faster compute
- ✓ Accelerated with sampling + parallel and distributed computing







Process graphs up to 4.1X larger on same hardware